College Entrance Examination Board

Advanced Placement Examination

MATHEMATICS: CALCULUS AB

SECTION II

Time—1 hour and 30 minutes

1. Let f be the real-valued function defined by $f(x) = \sqrt{1 + 6x}$.
   (a) Give the domain and range of f.
   (b) Determine the slope of the line tangent to the graph of f at $x = 4$.
   (c) Determine the y-intercept of the line tangent to the graph of f at $x = 4$.
   (d) Give the coordinates of the point on the graph of f where the tangent line is parallel to $y = x + 12$.

2. Given the two functions f and h such that $f(x) = x^3 - 3x^2 - 4x + 12$ and $h(x) = \begin{cases} \frac{f(x)}{x - 3}, & \text{for } x \neq 3 \\ p, & \text{for } x = 3. \end{cases}$
   (a) Find all zeros of the function f.
   (b) Find the value of p so that the function h is continuous at $x = 3$. Justify your answer.
   (c) Using the value of p found in (b), determine whether h is an even function. Justify your answer.

3. Let R be the region bounded by the curves $f(x) = \frac{4}{x}$ and $g(x) = (x - 3)^2$.
   (a) Find the area of R.
   (b) Find the volume of the solid generated by revolving R about the X-axis.

4. (a) A point moves on the hyperbola $3x^2 - y^2 = 23$ so that its y-coordinate is increasing at a constant rate of 4 units per second. How fast is the x-coordinate changing when $x = 4$ ?
   (b) For what values of k will the line $2x + 9y + k = 0$ be normal to the hyperbola $3x^2 - y^2 = 23$ ?

5. Given the function defined by $y = e^{\sin x}$ for all x such that $-\pi \leq x \leq 2\pi$.
   (a) Find the x- and y-coordinates of all maximum and minimum points on the given interval. Justify your answe
   (b) On the axes provided, sketch the graph of the function.
   (c) Write an equation for the axis of symmetry of the graph.

6. (a) Given $5x^3 + 40 = \int_c^x f(t)\,dt$.
      (i) Find $f(x)$.
      (ii) Find the value of c.
   (b) If $F(x) = \int_x^3 \sqrt{1 + t^{16}}\,dt$, find $F'(x)$.

7. For a differentiable function f, let f* be the function defined by $f^*(x) = \lim_{h \to 0} \frac{f(x + h) - f(x - h)}{h}$.
   (a) Determine $f^*(x)$ for $f(x) = x^2 + x$.
   (b) Determine $f^*(x)$ for $f(x) = \cos x$.
   (c) Write an equation that expresses the relationship between the functions f* and f′, where f′ denotes the usual derivative of f.

END OF EXAMINATION